

Thinking Mathematically

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From the Editor...

Working comfortably with negative numbers is essential for success in algebra, and many algebra students do poorly because they don't have a good grasp on dealing with negative numbers. Many of these students rely on trying to memorize rules, only to apply these rules incorrectly. For example, a student may remember that "two negatives makes a positive." While it is true that the *product* of two negative numbers is a positive number, this rule is often incorrectly applied to addition, where the *sum* of two negative numbers is negative.

I believe that it is important to begin acquainting students with negative numbers long before they are introduced in algebra, if possible. I also believe that rather than having students memorize the rules for adding, subtracting, multiplying, and dividing negative and positive numbers, it is better to allow them opportunities to discover these rules through games and activities.

I have seen young children quickly catch on to the rules for adding negative numbers when playing a card game in which players' scores can be negative. In the context of a game, the "rules" seem second nature.

I hope that you will find an activity or two in the following pages that helps your students master the operations with positive and negative numbers.

Focus Issue: Positive and Negative Numbers

Problem of the Month

Use each of the following digits exactly once to fill the nine squares so that each equation is true.

-1 -2 -3 -4 -5 -6 -7 -8 -9

$$\square + 1 = 0$$

$$4 - \square = 8$$

$$6 - 8 = \square$$

$$\square + \square = -10$$

$$\square \cdot \square = 30$$

$$16 \div \square = -2$$

$$101 - 110 = \square$$

Magic Squares

Try to fill in the missing squares. Each column, row, and diagonal must have the same sum. (Source: J. Weston Walch, 1983.)

| | | |
|----|----|----|
| 5 | -9 | |
| | -1 | |
| -3 | | -7 |

| | | |
|----|---|----|
| | 1 | 11 |
| 13 | 5 | |
| | 9 | |

| | | | |
|----|----|---|----|
| | -5 | | 6 |
| | 4 | 3 | |
| 2 | 0 | | 5 |
| -3 | | | -6 |

| | | | |
|----|----|----|----|
| 22 | -6 | | |
| | | 10 | |
| 8 | 4 | 2 | 14 |
| | 18 | 20 | |

What kind of number??

Students and teachers alike are often confused about the difference between natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers. Here's a brief summary.

Natural Numbers: $\{1, 2, 3, 4, 5, \dots\}$

Whole numbers: $\{0, 1, 2, 3, 4, 5, \dots\}$

Integers: $\{\dots-3, -2, -1, 0, 1, 2, 3, \dots\}$

Rational Numbers: Numbers that can be expressed as a fraction, $\frac{a}{b}$, where a

and b are integers and b is not equal to zero.

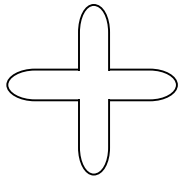
Irrational Numbers: any number that is a non-terminating, non-repeating decimal, like π or $\sqrt{2}$.

Real Numbers: All rational and irrational numbers.

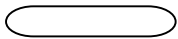
Hands-on Integers

Some students catch on to addition and subtraction of integers more quickly if they have opportunities to solve integer problems hands-on. The following activities involve using two different types of counters – one that represents positive integers and the other that represents negative numbers.

“Integer chips” that are sold commercially are one color on one side and one color on the other side. I prefer using “positive signs” and “negative signs” that help link the concept with the symbols. You can purchase inexpensive plastic tile spacers at hardware stores that look like this:



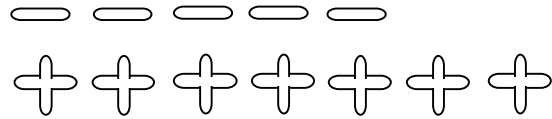
I cut off two pieces of it with a carpet knife to make the minus signs:



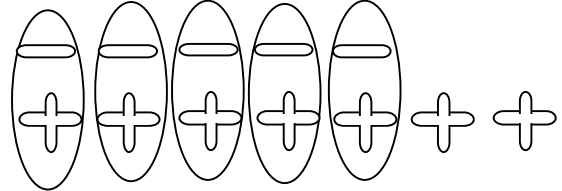
I made about 15 of each sign for each student.

You can use these pieces for several activities. To illustrate addition of integers, explain first to students that each + sign represents positive one, and each – sign represents negative one. Whenever you have a + and a – together, it is called a “zero pair”, equal to zero, and it can be removed from the problem.

To demonstrate $-5 + 7$, for example, put out five of the – signs and seven of the + signs, like this:



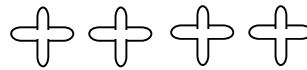
Then, remove all of the zero pairs:



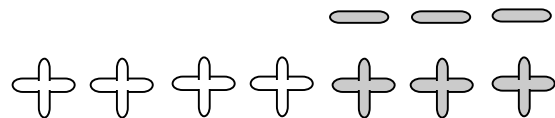
Whatever is left is your answer (positive two).

To use these pieces to illustrate the concept of subtraction of integers, follow this example:

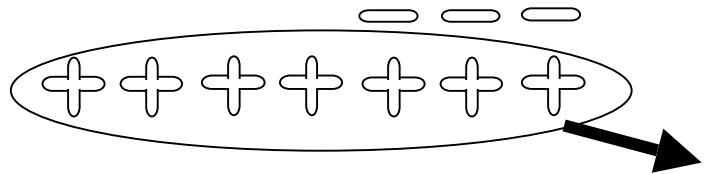
Suppose we want to subtract $4 - 7$. We begin by putting out four positive pieces:



We want to subtract a positive seven, but we only have positive four. So we add “zero pairs” like this:



Note that adding zero pairs doesn’t change the value. Now we can subtract seven positive pieces:

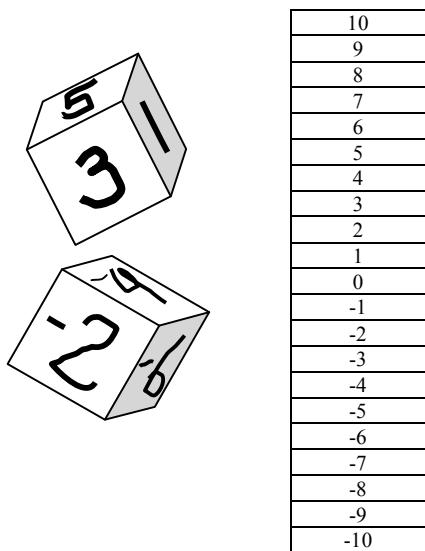


And what is left is the answer: $4 - 7 = -3$.

To see more examples, you can visit the GREAT web sites on page 5!

Race to 10

Race to 10 is a game I have used many times to help students internalize the rules for adding positive and negative numbers. To play this game, make an enlarged version of the game board below for each pair of students.



Also, you need to make two number cubes for each pair of students. The most inexpensive way to make these is to use wooded cubes, available at craft stores. Write 1, 2, 3, 4, 5, and 6 on one die and -1, -2, -3, -4, -5, and -6 on the other die.

Each student places a marker of some kind (I usually use two different colored paper clips) at zero. Students take turns rolling the two dice and adding the results. The answer tells them how many spaces to move up (positive result) or down (negative result).

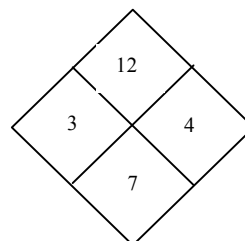
For example, if Player A rolls a -3 and a 5, the sum is 2, so the player moves two spaces up. On her following turn, if player A rolls a 1 and a -6, the sum is -5, so the player moves down 5 spaces, ending up at -3.

Players take turns. The first player to get off the board either in the positive direction or the negative direction wins.

Diamond Problems

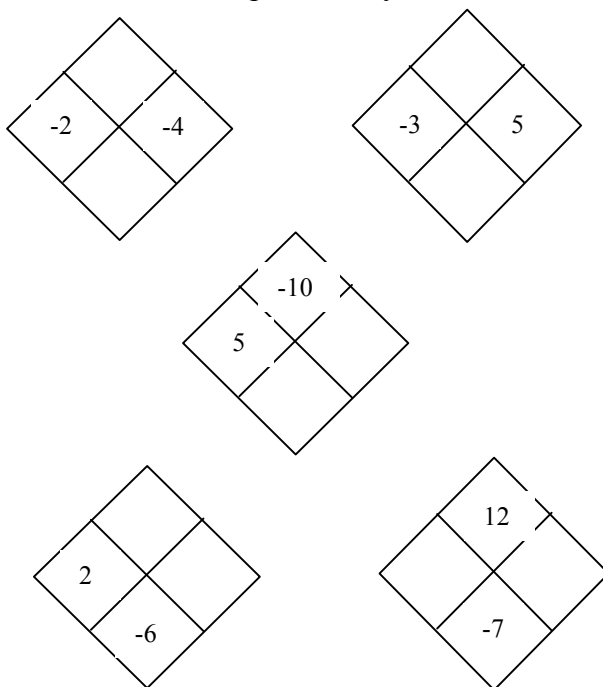
This activity comes from *Algebra Connections* (CMP – College Preparatory Mathematics, Kysh, Sallee, Dietiker, and Hoey). These puzzles can be made using whole numbers, fractions, integers, algebraic expressions – whatever your students are working on.

Here is an example of a Diamond Problem:



Do you see the relationship among the numbers? (The top number is *product* of the two middle numbers, and the bottom number is the *sum* of the two middle numbers.)

Have your students see if they can fill in the missing spaces in these Diamond Problems. Make up more of your own!



Answers

Problem of the Month, p. 1

$$\boxed{-1} + 1 = 0$$

$$4 - \boxed{-4} = 8$$

$$6 - 8 = \boxed{-2}$$

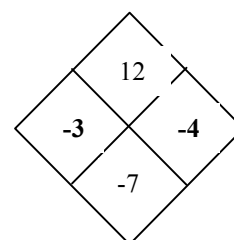
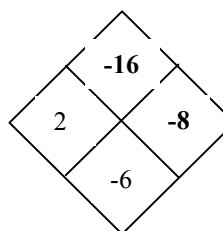
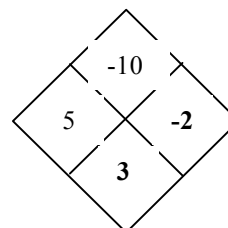
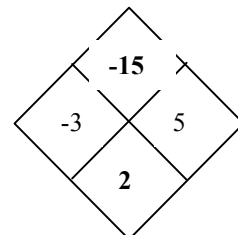
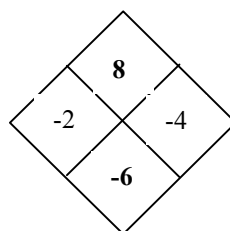
$$\boxed{-3} + \boxed{-7} = -10$$

$$\boxed{-5} \cdot \boxed{-6} = 30$$

$$16 \div \boxed{-8} = -2$$

$$101 - 110 = \boxed{-9}$$

Diamond Problems



Magic Squares

| | | |
|-----------|----------|----------|
| 5 | -9 | 1 |
| -5 | -1 | 3 |
| -3 | 7 | -7 |

| | | |
|-----------|---|-----------|
| 3 | 1 | 11 |
| 13 | 5 | -3 |
| -1 | 9 | 7 |

| | | | |
|-----------|----------|-----------|----------|
| 9 | -5 | -4 | 6 |
| -2 | 4 | 3 | 1 |
| 2 | 0 | -1 | 5 |
| -3 | 7 | 8 | -6 |

| | | | |
|-----------|-----------|-----------|-----------|
| 22 | -6 | -4 | 16 |
| 0 | 12 | 10 | 6 |
| 8 | 4 | 2 | 14 |
| -2 | 18 | 20 | -8 |

WEB SITES WITH NEGATIVE NUMBER ACTIVITIES

http://nlvm.usu.edu/en/nav/grade_g_3.html

Scroll down the list to see “Color Chips – Subtraction”. The “Circle 0”, “Circle 21”, and “Circle 3” games all give great practice with integer addition. This is a fantastic web site – take the time to browse through all it has to offer.

<http://mathforum.org/mathtools/ce/m7,9.6,ALL,ALL/>

The top activity on the list uses integer chips, but all are good – check them out!