Thinking Mathematically

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From the Editor…

Patterns are at the heart of mathematics. When students become better at seeing patterns, figuring out what comes next in a pattern, and generalizing a pattern, they will become better at mathematics.

There are many types of patterns, including visual patterns and number patterns. Patterns are like puzzles – "figure out what comes next" – and students usually enjoy them.

Patterns are also a topic that is often neglected in math classes because there is rarely a "unit" or "chapter" devoted to the study of patterns. This means that they need to be brought into lessons as an "extra". But time spent discussing and studying patterns is valuable and very worth while, as it helps students use their higher order thinking skills and to generalize and see the whole picture. (And yes, there are patterns on the GED!)

I hope you enjoy the patterns in this issue. There are some fun "recreational" patterns, some basic "what number comes next" patterns, as well as more involved "what would the 50th figure in this pattern look like?" type patterns.

If any of you have some interesting patterns I can include in an upcoming issue, please send them my way!

Thanks,

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Focus Issue:

Patterns

Problem of the Month

Can you figure out the patterns below and fill in the blanks? (answers p. 5)

1, 2, 4, __, __, __

J31, F28, M31, A30, __, __, __

O, T, T, F, F, S, S, __, __, __

77, 49, 36, 18, ____.

Check out these web sites!

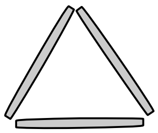
http://www.learner.org/teacherslab/math/patterns/mystery/ - Mystery Number Game

http://cybersleuth-kids.com/sleuth/Math/Math_Worksheets/Number_Patterns/index.html - Printable number pattern worksheets

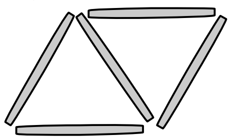
http://www.funbrain.com/cracker/ - On-line fun "find the next number" game

Toothpick Patterns

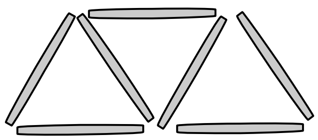
Make the following triangle out of toothpicks:



Then add two toothpicks to make two triangles:

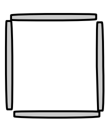


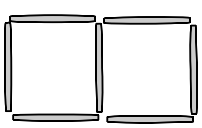
And again, making three triangles:

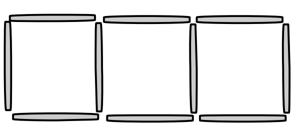


Ask students how many toothpicks they would need to make a chain of 10 triangles (21). How about if you wanted to make a chain of 50 triangles? (101) What is the rule? (Multiply the number of triangles by two, and add one.)

A similar problem can be posed with squares made out of toothpicks:







How many toothpicks are needed to make a chain of 13 squares? (40) And for 60 squares? (181) What is the rule? (Multiply the number of squares by 3 and add 1.)

In/Out Tables

Determine the rule for each in-out table below. Then fill in the missing outputs. (answers page 5)

1.

IN	OUT
0	0
1	3
2	6
3	9
4	12
5	?
134	?

2.

IN	OUT
0	4
1	5
2	6
3	7
4	8
5	?
247	?

3.

IN	OUT
0	1
1	3
2	5
3	7
4	9
5	?
163	?

4.

IN	OUT
0	-1
1	1
2	3
3	5
4	7
5	?
107	?

5.

IN	OUT
0	0
1	½
2	1
3	1 ½
4	2
5	?
672	?

6.

IN	OUT
0	1
1	5
2	9
3	13
4	17
5	?
365	?

Make up your own in-out tables for your students.

Addition Table Patterns

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

The addition table pictured above is full of wonderful patterns. Some patterns are more obvious than others. See how many patterns your students can find in this grid. Some are listed below.

- Diagonals going in one direction (up, from left to right) are all the same number
- Diagonals going in the other direction (down, from left to right) are alternately even numbers and odd numbers
- If you draw a square around any nine numbers in the table, the sum of the two diagonals is the same as the sum of the middle three numbers going across and going down. For example, in this square section of the grid, the two diagonals add to 12, and the center column (3 + 4 + 5) and center row (3 + 4 + 5) also add to 12.

2	3	4
3	4	5

- In any square of nine numbers in the table, such as

3	4	5
4	5	6
5	6	7

the middle number is always the mean (average) of all the numbers in the square.
- If you double the middle number in any square of nine numbers, you get the sum of any pair of opposite corners.
- Furthermore, if you add the four corners and divide by the middle number, it is always equal to 4.
- If you add the eight numbers around the outside of a square of nine numbers and divide by the middle number, it is always equal to 8.
- If you draw a rectangle around any portion of the grid, the sum of the opposite corners is always equal. For example, in this rectangular portion of the grid, 4 + 11 = 7 + 8.

4	5	6	7	**8**
5	6	7	8	9
6	7	8	9	10
7	8	9	10	**11**

- If you look at a 4 by 4 piece of the grid, such as this:

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

You find that the sum of the four numbers in the middle is always equal to 1/3 of the outside ring.

PRETTY COOL, EH?

Generalize the Pattern

These patterns can be used with all different abilities. Basic math students can be asked to describe the pattern and draw what comes next.

To make the problem more challenging, students can fill in the chart up through Stage 6 and see if they see a pattern in the numbers in the second column.

And algebra students can be asked how many pieces are in stage 32, for example (or any stage that is too big for them to actually draw) and then to give the general rule for the n^{th} stage.

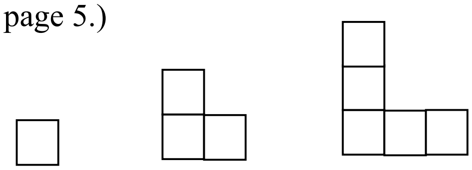
For each pattern I present to algebra students, I give them the following directions and chart:

*For each pattern, draw **diagrams** of the first few stages, fill in the chart, and then write a **description** of the pattern. Describe how you get from one stage to the next, and describe how you can find the general rule for any stage number.*

Stage number	Number of pieces
1	
2	
3	
4	
5	
6	
10	
32	
n	

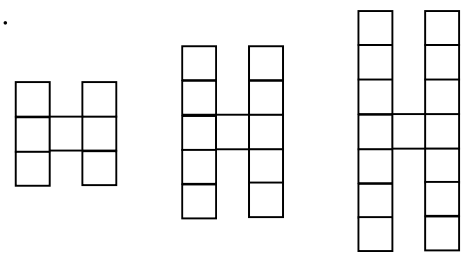
Here are some patterns. See if you can discover the rule and the algebraic expression for the n^{th} stage. (Answers are on page 5.)

1.



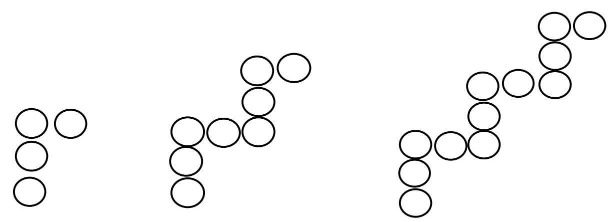
stage 1 stage 2 stage 3

2.



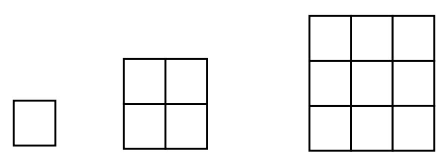
stage 1 stage 2 stage 3

3.



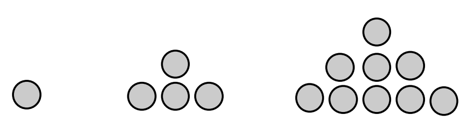
stage 1 stage 2 stage 3

4.



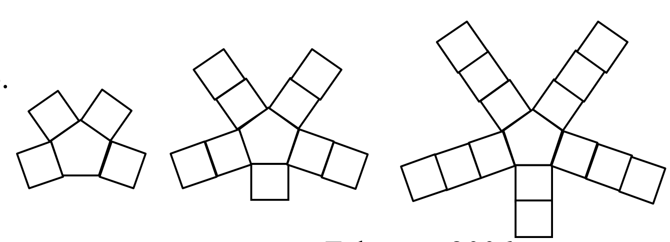
stage 1 stage 2 stage 3

5.



stage 1 stage 2 stage 3

6.



stage 1 stage 2 stage 3

Answers

Problem of the Month, p. 1

1, 2, 4, 8, 16, 32
1, 2, 4, 7, 11, 16
This pattern could be doubling (top answer) or it could be +1, +2, +3, etc (bottom answer).

J31, F28, M31, A30, M31, J30, J31
The letters are the first letters of the months, and the numbers are the number of days in the month. For example, the first item in the pattern, J31, represents January having 31 days.

O, T, T, F, F, S, S, E, N, T
These are the first letters of the counting numers: One, Two, Three, etc.

77, 49, 36, 18, 8 .
Multiply the two digits in each number to get the next item in the pattern. For example, 7×7 = 49, and 4×9 = 36. This pattern ends with the last term, 8.

In/Out Tables

1. 5 → 15
   134 → 402
   Rule: Multiply by 3

2. 5 → 15
   247 → 402
   Rule: Add 4

3. 5 → 11
   163 → 327
   Rule: Double the IN number and add 1.

4. 5 → 9
   107 → 213
   Rule: double the IN number and subtract 1.

5. 5 → 2 ½
   672 → 336
   Rule:Divide by 2 (or multiply by ½)

6. 5 → 19
   365 → 1459
   Rule: Multiply by 4 and subtract 1.

Generalize the Pattern, p. 4

1. Generalized rule: If you double the stage number and subtract 1, you get the number of squares. (2n – 1)

2. If you multiply the stage number by 4 (because there are 4 "legs" in the figure) and add 3 (for the 3 pieces across the middle), you get the total number of squares. (4n + 3)

3. In each stage, we add 4 circles, so to get the number of circles, multiply the stage number by 4. (4n)

4. This pattern represents the square numbers. $1^2 = 1$, $2^2 = 4$, etc. So the number of tiles in each stage is the stage number squared. (n^2)

5. This is another way to represent square numbers. The number of circles in each stage is the stage number squared. (n^2)

6. The center pentagon remains constant, and 5 squares are added at each new stage. (5n + 1)