

Thinking Mathematically

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From the Editor...

As I sit down to compile activities for this issue on Algebraic Thinking, I realize that just about every issue so far could be considered algebraic thinking, or at least preparing for algebra. Positive and Negative numbers, number sense, and especially patterns could all fall into the category of algebraic thinking.

More and more teachers realize the importance of teaching students to “think algebraically”, and algebraic thinking activities are being introduced in the lower elementary grades now, whereas algebra used to be the realm of high school only.

When you ask students to find a general rule, they are thinking algebraically. In the last issue about patterns, the activity called “Generalize the Patterns” contains examples of taking a specific situation and generalizing it algebraically. In my experience, students really enjoy these “puzzles” and pick up on the patterns quickly.

This issue contains more such activities, at various ability levels, so whether you teach basic education or high school algebra, there will hopefully be an activity that you will be able to use.

~Ruth

Focus Issue: Algebraic Thinking

Problems of the Month

::: ROBOTS :::

Two robots are strutting through cyberspace, trying to keep in touch with everything on the World Wide Web.

Abot says to Bbot, “I get so many headaches trying to listen to so many radio signals at the same time! Do you know that if you gave me two of your antennas we’d have the same number?”

Bbot retorted, “That’s nothing! If I had two of your antennas, I’d have five times as many as you!”

How many antennas does each of them have?

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Brothers and Sisters...

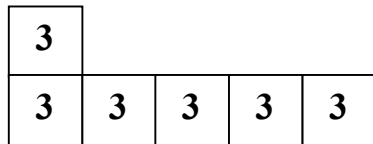
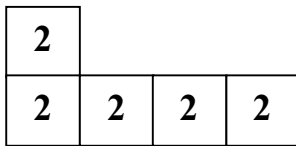
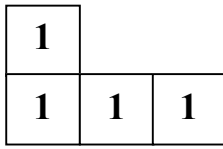
Jennifer’s family has fewer than 10 children. She has the same number of sisters as she has brothers, but her brother Michael has twice as many sisters as he has brothers. How many boys and how many girls are in the family?



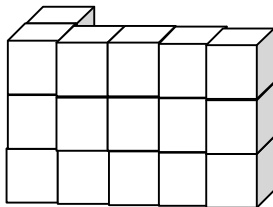
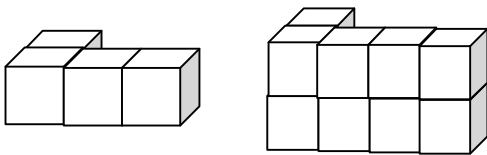
Block Buildings

Nelly and Omar are building a sequence of block buildings. The first three are shown in the diagrams below. The number shows how many blocks are stacked up on each spot.

How many blocks are needed for the fourth building? The tenth building? The n th building? (solution on page 5)



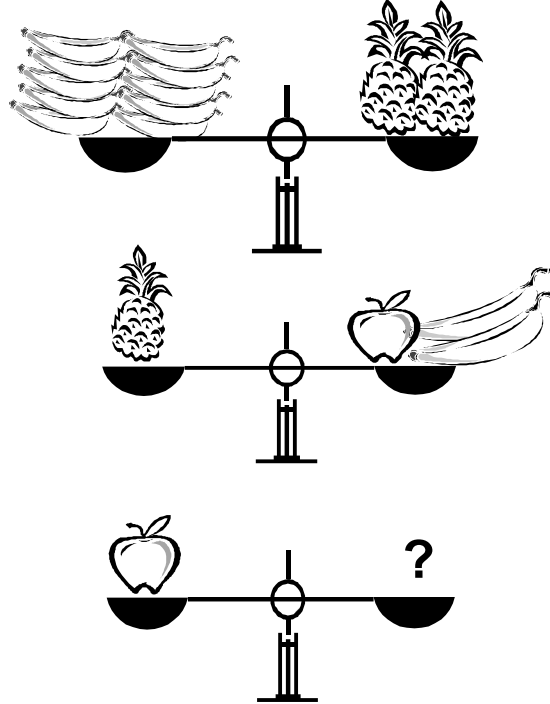
Here is what these first three buildings look like in 3-D:



[source: NCTM's Mathematics Teaching in the Middle School, March 2003.]

Bananas

How many bananas do you need to make the third scale balance? Explain your reasoning.



[Source: Britannica Mathematics Systems]

Going Backwards

Pat and Chris are playing a game. One player writes down an arrow string and the answer, but not the starting number. The other player has to determine the starting number.

Here is pat's arrow string and answer:

$$\underline{\quad} \xrightarrow{+4} \underline{\quad} \xrightarrow{\times 10} \underline{\quad} \xrightarrow{-2} \underline{\quad} \xrightarrow{\div 2} 29$$

What should Chris give as the starting number? Here is Chris's arrow string and output.

$$\underline{\quad} \xrightarrow{+3} \underline{\quad} \xrightarrow{+6} \underline{\quad} \xrightarrow{+5} \underline{\quad} \xrightarrow{-2} 6$$

What should Pat give as the starting number?

Source: Britannica Mathematics Systems.

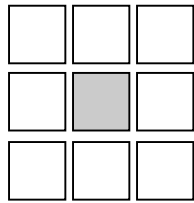
Garden Beds

Materials:

About 5 yellow tiles to use as “garden beds” and about 16 green tiles to use as “borders”.

A gardener wants to place a tile border around a flowerbed.

Surround a flowerbed of one square with tiles. How many do you need?



Now build a border around a flowerbed the size of two squares. How many tiles do you need?



Try building borders around flowerbeds of 3, 4, and 5 squares (Flower beds are always one tile wide). Each time record the number of tiles you need in a table like this:

Size of flowerbed	Number of tiles
1	
2	
3	
4	
5	

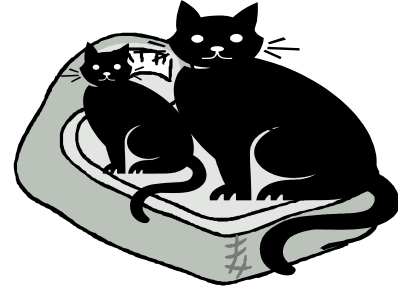
Your challenge is to predict the number of tiles that would be needed for any size flowerbed, For example, what if the garden bed is 100 tiles long?

Source: Curriculum Corporation, 1995

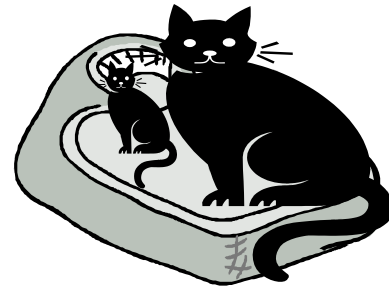
Cats

Lou has three cats. One is small, the other is medium size, and the third one is large. The cats weighed themselves in different groupings like this:

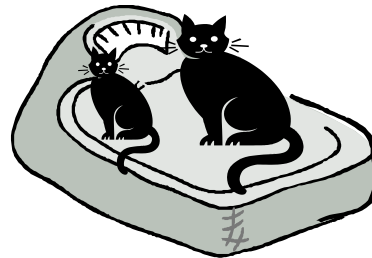
The medium cat and the big cat weigh 14 pounds.



The small cat and the big cat weigh 11 pounds.



The small cat and the medium cat weigh 7 pounds.



What do all three cats weigh together? What does each cat weigh? Show how you got your answer.

Lining Up

Materials:

37 index cards or markers to represent people – one is different from the rest



1. A group of people are lining up in one line. You are 15th from each end. How many people are in your class?
2. What if you were 7th from each end?
3. What if you were 3rd from each end?
4. Can you find a rule for working out the answer if you are n^{th} from each end? In each case, what sort of number is the number of people in the line?

The Big Race

Al, Bob, Carl, Doug, and Ed are in a race. Halfway through the race, they are in these positions: Al is 20 yards behind Bob, Bob is 50 yards ahead of Carl, Carl is 10 yards behind Ed, and Doug is 30 yards ahead of Al. Who, at this point, is winning the race? Who is second? Who is third?

[source: NCTM *Mathematics Teaching in the Middle School*, Nov. 2003]



To Succeed in Algebra...

The following skills are important to have before beginning algebra.

- Number Sense
 - Understanding the properties of numbers
 - Being able to use all numbers, not just whole numbers
 - Recognizing patterns
- Operation Sense
 - Seeing the operation (relationship), not just the answer
 - Understanding the equals sign (it means that the two sides have the same value; it does NOT mean “give the answer”)
- Symbol Sense
 - Being able to generalize patterns
- Expression Sense
 - Being able to translate English into mathematical language and vice versa
- Graph Sense
 - Familiarity with coordinate plane

[source: Dr. Judy Curran Buck, Plymouth State College, November 2000 workshop]



Answers

Brothers and Sisters, p. 1

There are 4 girls and 3 boys.

Robots, p. 1

Abot has 4 antennas and Bbot has 8 antennas.

Block Buildings, p. 2

The fourth building needs 28 blocks and the tenth building needs 130 blocks.

For each stage, the number of blocks in the “footprint” is $n + 3$, and the building is n blocks high, so the n th building requires $n(n + 3)$ blocks.

Bananas, p. 2

If 10 bananas weigh the same as 2 pineapples, then 5 bananas weigh the same as 1 pineapple. The second picture shows 1 pineapple weighing the same as 2 bananas and an apple, **so the apple must weigh the same as 3 bananas.**

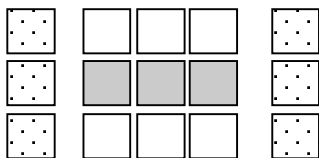
Going Backward, p. 2

The first arrow string begins with 2 and the second arrow string begins with 15.

Garden Beds, p. 3

Size of flowerbed	Number of tiles
1	8
2	10
3	12
4	14
5	16
n	$2n + 6$

For any size garden bed, you have twice the number of tiles in the garden bed, for the top and bottom (white tiles in the following figure), plus the three tiles on the left and the three tiles on the right (polka dotted tiles), giving us $2n + 6$.



So for a garden bed that is made up of 100 tiles, you would need 206 tiles for the border.

Cats, p. 3

The small cat weighs 2 pounds, the medium cat weighs 5 pounds, and the big cat weighs 9 pounds.

One way to discover this is to add all three pictures, with their weights, together, so you find that 2 of each size cat weighs 32 pounds so one of each cat weighs 16 pounds. If all three weigh 16 pounds, and the medium and large cat together weigh 14 pounds, then the small cat must weigh $16 - 14 = 2$ pounds. Use the other two pictures to figure out the weights by subtracting.

Lining Up, p. 4

- 29 people
- 15 people
- 5 people
- If you are n people from each end, then there are $2n - 1$ people in the line. There will always be an odd number of people in the line.

The Big Race, p. 4

Doug is in first place, Bob is in second, Al is in third, Ed is in fourth, and Carl is in last place.