Mindfulness and meditation
Mindfulness is practicing bringing awareness to what is happening now with the intent to be interested and non-judgmental. Mindfulness is a quality of being in the world that we cultivate over time. Practicing mindfulness is like taking the temperature of the mind-body system: what’s going on now, beginning with awareness of the breath and body. We are not trying to stop the mind but rather tuning in to how it is.

- **practicing**: Practicing reminds us that our ability to be more mindful grows over time.
- **now**: The focus is on present moment experience vs. the past or future.
- **intention**: Intention lets go of ‘should’ and ‘expectation’ and ‘doing it right.’
- **curious**: Think of the curiosity of a 3-year old child or a scientist.
- **non-judgmental**: Judgmental is “I like/want” and “I don’t like/want.” It clouds our vision. When we are aware of judgments, we see more clearly.
  Non-judgmental connects to Avatar: “I see you” and to “Namaste.”

Meditation is when we practice mindfulness for a period of time: while sitting, lying down, standing, or walking; focusing on the breath, the body, thoughts/emotions, sounds, sights or with an open awareness.

The goal of mindfulness meditation is not to make the mind quiet or to have a peaceful meditation, though that is great when it happens. This practice is about getting the mind to be more aware of the present moment so that we can see our reality more clearly. In doing so, we increase those times where we respond to events rather than react to events.

Outcomes of mindfulness (from research and Tom’s experience): reduced stress, increased joy, better relationships, tools to deal with life’s issues—all of these make for more engaged students and the classroom as a community.

Meditations:
- **Breath**: Some prefer a zoom lens—where you feel the breath most clearly—nostrils, abdomen, chest, elsewhere. Some prefer a wide angle lens—what do you notice about each breath; can include qualities of the breath—shallow/deep, fast/slow, rhythmic/choppy.
  If tired, you can focus more on the in-breath—oxygen literally coming into your system.
  If restless or anxious, focus more on the out-breath which is naturally relaxing.
- **Body**: A common practice is the body scan: moving systematically through the body, paying attention to sensations or the energy that you notice in each part of the body.
- **Sounds**: The focus here is still on direct experience, that is, the intention to be less conceptual.

Guided meditations by Tom and by Tara Healy can be found on tom-bassarear.squarespace.com
Factors affecting how one constructs knowledge: Students' beliefs about intelligence

What follows here is Carol Dweck’s original model. Since then (when I was in graduate school), she has written an excellent book called *Mindsets* where she has modified her original work. I still lean toward her original model when thinking about learners.

Carol Dweck and Elaine Elliott proposed that students hold, *to differing degrees*, two operating conceptions of intelligence which, in turn, lead to two different achievement goals. Students’ conceptions of the nature of intelligence often strongly influence the goals which they seek and the persistence with which they pursue those goals. The table below summarizes some of the salient differences between the two theories of intelligence and the consequent achievement goals and tendencies.

<table>
<thead>
<tr>
<th>Theories of Intelligence</th>
<th>Incremental</th>
<th>Entity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intelligence is:</td>
<td>A repertoire of skills that increases through effort.</td>
<td>A global, stable entity whose adequacy is judged through performance.</td>
</tr>
<tr>
<td>Effort is:</td>
<td>An investment that increases intelligence.</td>
<td>A risk that may reveal low intelligence</td>
</tr>
<tr>
<td>Goals</td>
<td>Competence Increase</td>
<td>Competence Judgment</td>
</tr>
<tr>
<td>Entering questions:</td>
<td>What will I learn?</td>
<td>How will I do?</td>
</tr>
<tr>
<td>Focus on:</td>
<td>Process</td>
<td>Outcome</td>
</tr>
<tr>
<td>Errors viewed as:</td>
<td>Natural, useful</td>
<td>Failure</td>
</tr>
<tr>
<td>Uncertainty and ambiguity viewed as:</td>
<td>Challenging</td>
<td>Threatening</td>
</tr>
<tr>
<td>Teacher's role seen as:</td>
<td>Resource, guide</td>
<td>Judge, rewarder/punisher</td>
</tr>
</tbody>
</table>

Students subscribing to an entity view of intelligence tend to see intelligence as a rather stable, global trait (e.g., you either have it or you don't). Such students tend to believe that they possess a specific, rather fixed amount of intelligence. Furthermore, they feel that this intelligence is displayed through performance and that performance is judged by them and by others to reflect their level of their intelligence. On the other hand, students subscribing to an incremental view of intelligence tend to see intelligence as consisting of a repertoire of skills that can be increased through one's own actions.

According to Dweck and Elliot's model, different theories of intelligence lead to different achievement goals—learning or performance. Some students are motivated predominantly by learning goals. Their focus is primarily on increasing their competence, seeking to master or understand new knowledge or skills. Other students are motivated more by performance goals. Such students tend to focus more on obtaining favorable judgments of their competence and on avoiding unfavorable judgments of their competence, i.e., obtaining a high grade or avoiding a low grade. These different achievement goals (learning or performance) lead students to structure the same achievement situations in very different ways. It is hypothesized that students motivated predominantly by performance goals are more vulnerable to maladaptive behaviors in the face of failure.
Factors affecting how one constructs knowledge: Attribution theory

An entertaining and persuasive introduction to attribution theory comes from a "Dear Abby" column which was printed under the headline: She can't see beyond her nose.

Dear Abby:
I am a 34-year-old woman who has divorced three husbands. (Not my fault. I always picked losers.) My problem is my nose. I had plastic surgery on it when I was 18, and the doctor botched the job, so at 21 I had it reshaped and then it was worse. I think it makes me look stuck up and keeps me from making friends. I went to a well-known plastic surgeon, and I offered to pay him in full in advance but he refused to take me as a patient! He said he didn't think any plastic surgeon could please me because I had "emotional and social problems" I should face up to instead of blaming everything on my nose. Then he insulted me further by suggesting that I use my money to see a psychiatrist! Abby, there is nothing wrong with my mind. It's my nose! Will you please recommend a good plastic surgeon? I can afford to go anywhere.

Determined in Hartford

This letter brings to mind any number of students having trouble in classes whose attributions for their failure (e.g. "the teacher doesn't like me," "the test was unfair") are as much a part of their failure as any cognitive deficits. In fact as long as the attributions remain so maladaptive, a teacher's attempts to get the student to study more, to try harder, to pay better attention, etc. are likely wasted.

Attribution theory holds that when persons succeed or fail at a task they tend to attribute the success or failure to various factors, for example, their effort, their ability, luck, or help from others. It is important to note that the assessment often takes place below a level of immediate awareness. These attributions, especially in the face of failure, prompt a search for the cause of the outcome.

Proponents of attribution theory argue that it is not success or failure per se but rather the causal attributions made for these outcomes that influence future expectancies and behavior. It is assumed that the manner in which one interprets outcomes guides the hope of subsequent success and thereby influences subsequent achievement-related behaviors (e.g., choice, persistence, etc.) The table below operationalizes some essential aspects of this theory. For example, a student attributing failure on a test to not going to the tutor for help (controllable factor) is more likely to study harder the next time than a student attributing failure to low ability (uncontrollable factor).

<table>
<thead>
<tr>
<th>Success</th>
<th>Controllable</th>
<th>Uncontrollable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal</td>
<td>I did well because I studied hard.</td>
<td>I did well because I'm smart.</td>
</tr>
<tr>
<td>External</td>
<td>I did well because teacher helped me.</td>
<td>I did well because lucky; the teacher likes me.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Failure</th>
<th>Controllable</th>
<th>Uncontrollable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal</td>
<td>I didn't do well because I didn't study hard enough; didn't study correctly</td>
<td>I didn't do well because I just don't have it; because I hate the teacher/subject.</td>
</tr>
<tr>
<td>External</td>
<td>I didn't do well because I didn't get enough help.</td>
<td>I didn't do well because I was unlucky; the teacher doesn't like me</td>
</tr>
</tbody>
</table>
Over the years, I have found a strong correlation between students who suffer from more anxiety and students who are more focused on performance goals. Anxiety is not a simple concept like pregnancy: you are either pregnant or not pregnant. I offer anxiety as a multi-dimensional construct.

First, we must distinguish between facilitative and debilitative anxiety, because anxiety itself is not "bad." If there is a test and you know that you do not really understand the basic ideas, then anxiety is a helpful emotion--it will prompt you to take action.

Second, we must distinguish between a cognitive and an emotionality component of anxiety. Worry, the cognitive component, is seen as cognitive concern about one's performance whereas the emotionality component is seen as arousal of the autonomic nervous system in evaluative situations. Interestingly, a consistent relationship has been found in studies between worry and performance while no consistent relationship has been found between emotionality and performance.

Third, we must distinguish between two types of anxiety--trait and state (school, math, test). Many people who suffer from math anxiety do not ordinarily exhibit such anxiety in other achievement situations.

Fourth, we must distinguish between task-irrelevant thoughts and task-relevant thoughts. Anxious students who are doing poorly tend to have a much higher percentage of the former kinds of thoughts. That is, in five minutes of working on a problem, only a small portion of thought is relevant to the task; the rest of it is irrelevant to the task, e.g., "I can't believe he gave us this problem, and there's still 30 minutes left, and I have so much homework to do; I hope I don't have to take this class over; this is a stupid problem anyway..."

The causes of a learner's anxiety suggest different interventions.

1. Anxious in general
   Some students are anxious in general, not just in class. I often encourage such students to seek help at the counseling center.

2. Anxious in school
   Some students are anxious in school but not outside school. Some of these students don't see themselves as academic, for example, students who have been pressured to go to college by their parents though their heart is not in it, or older students who are coming back to school after some time. Depending on the cause of the anxiety, I encourage such students to seek help at the math center, or a student skills support program, or even career counseling; in other cases, I try to help them see the downside of obsessing about a grade.

3. Anxious in math courses
   Some students are anxious in math courses, but not in their other courses. Many of these are conscientious students whose learning style is different from the traditional mathematics teacher, e.g., inductive learner, concrete learner, learns best in a context-rich environment. If I suspect a genuine learning disability, I refer them to another office on campus. However, in many of these cases, I offer a reframing of their problem: they not ‘learning disabled’ as much as the system and former teachers have been ‘teaching disabled.’ That is, the teachers' teaching style didn't honor their learning style.

4. Anxious on tests
   Some students are anxious only in test situations. I have had great success by offering alternative testing procedures. If the student is truly anxious, enabling him to take the test in a quiet room or taking extra time solves the problem. On the other hand, if that is not the primary cause (e.g., the denial student), the failure of this remedy enables us to focus more on the actual causes.
Five kinds of students

I offer five different 'types' of vulnerable students, each of which calls for different responses and strategies on the part of the teacher. I elaborate on the first three types below. The helpless student--entity conception of intelligence, low confidence, and attributions for success and failure to uncontrollable factors. The 'denial' student--low skills yet high confidence. The pressured student--high anxiety and high perceived usefulness of math. The 'don't care' student--low perceived usefulness, low anxiety, and who usually 'hates' math. The naive student--low scores on the beliefs variables; in other words, a student who pretty much sees learning as memorizing and regurgitating.

The helpless student essentially has little or no confidence that his/her effort alone will be sufficient to learn the material. Such a student often gives up very easily, after only minimal effort. Helpless students often make statements like the following. "I'm just no good in math." "My mind just goes blank." "The only way I'll pass the class is by getting lots of help." There is a rich literature in psychology on learned helplessness and a growing literature in mathematics education on helplessness.

The most succinct description of the second type of student is denial. There seem to be two basic types of denial in students. Some students insist that they really know the material but just make "lots of little mistakes." For example, one of my students said he thought his grade at the time of the interview was "either a low C or a high D." When asked what his strengths were as a math student, he replied that he really knew the basics. In fact, he had received a 25% on the first exam and had not passed any of the five chapter quizzes. Another pattern of denial is referred to in the psychology literature as "defensive attributions." Following are several statements such a student is likely to make: "I didn't feel good on the day of the test." "I can't learn from this teacher (this book, this system, etc.)." "I didn't have enough time to do the test (to do the assignment)." As with helplessness, there is a rich psychological literature pertaining to denial.

At the heart of both helplessness and denial is the desire to preserve one's self-esteem. The helpless student truly feels stupid, at least in math. The act of giving up, the helpless posture, can be seen as "cutting one's losses." Since the student feels that perseverance won't pay off anyway, by not seriously engaging with the material, the helpless student at least reduces the frequency and intensity of times of feeling so stupid. The denying student, on the other hand, refuses to acknowledge his/her lack of mathematics skills. Should such a student do poorly in the course, the student has a number of alternative explanations rather than "I cannot learn this material."

A third type of student I have called pressured. This includes the classic "math anxious" student but goes deeper into the causes of anxiety, which often have to do with either having to get a good grade in the course or needing to master the material in the course for some specific reason. The pressured student is likely to make statements like the following. "I have to get at least a B in this class." "I've got to learn this stuff because I have to take statistics next semester." "I've got to have a high GPA for graduate school." Pressured students are generally "answer" and "grade" oriented. The analogy to a horse with blinders come to mind. Just as that horse cannot see the surrounding environment, the pressured student misses many cues in the mathematical environment (e.g., similarities from one problem to another and heuristics which can be applied to many types of problems) because s/he is so preoccupied with getting the answer, applying the algorithm, or getting the desired grade.

Although most of a teacher's attention should necessarily be in the direction of increasing the student's cognitive and metacognitive skills, focusing on these types can enable teachers also to address the psychological-motivational factors which impede such students' ability to learn effectively.
Maehr asserts that meaning is the critical determinant of motivation. Each student comes to the classroom with a package of meanings derived from past experiences. Whether or not the students will invest themselves in a particular activity depends on what the activity means to them. According to Maehr, the extent to which students will invest themselves depends on several interrelated factors:

1. Judgments about self, especially beliefs about one's competence or ability to master the material,
2. Their perceived goals,
3. The subjective cost of success
4. Judgments about the options or alternatives available for reaching these goals.

These factors have been characterized with respect to the issues and questions they raise:

<table>
<thead>
<tr>
<th>Issue</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Competence</td>
<td>Can I do it?</td>
</tr>
<tr>
<td>2. Goals</td>
<td>What will I get out of this activity?</td>
</tr>
<tr>
<td>3. Cost</td>
<td>Will it be worth the effort?</td>
</tr>
<tr>
<td>4. Autonomy</td>
<td>Will it be my learning?</td>
</tr>
</tbody>
</table>

The desire for competence has long been established as an essential motivating element in the learning process. This was brought home to me during my first year of teaching. So many of my pre-algebra and algebra 1 students who plaintively asked, "Why do we have to learn this?" stopped complaining when they were able to master the material.

With respect to autonomy, as the individual sees herself as the initiator or cause of her own behavior, she will more likely find her own reasons for engaging more deeply in the activity. This is one of the reasons for more attention to "real" problems in mathematics classes nowadays.

A high degree of personal investment produces a student characterized by continuing motivation, as opposed to the more common stop-start cycle of motivation exhibited by many students. It is important that the teacher focus on nurturing and developing continuing motivation, for this kind of motivation is not commonly seen in traditional classrooms and is crucial in the constructivist classroom with its higher focus on problem solving and other higher order thinking skills.