How Attitudes and Beliefs Can Affect A Student’s Success in Mathematics
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6 Important Variables

(1) Conceptual Knowledge
The depth of a student's understanding of content matter is obviously a powerful factor. For example, there is a world of difference between merely memorizing the rules for multiplying and dividing decimals and being able to explain why the procedures work.

(2) Computational Ability
Although these first two are strongly connected, I have separated this from knowledge of content because I have observed so many cases in which a marginal student's limited computational skills have interfered with his/her ability to learn new concepts.

(3) Problem Solving Knowledge
There are a number of problem solving strategies which one can use while solving problems. It is helpful to separate out two components: a need to have many such strategies in one's repertoire and a need to be able to use them skillfully.

(4) Metacognitive Knowledge
These generally involve being able to monitor one's progress while solving the problem, for example, realizing when one is on shaky ground and needs to reread the problem or needs to try another strategy, realizing when one’s attention is waning or wandering, etc.

(5) Beliefs about Mathematics and about Learning
These can include beliefs about mathematics, about problem solving, about learning mathematics, about the teacher's role (e.g., s/he is supposed to show me how to do it), about the learner’s role, and about the causes of their success and failure. Some examples:
Beliefs about problem solving
  “You’re supposed to do it mathematically, e.g., find the formula.”
  “If you can’t get the answer right away, forget it, you never will,” or “you’re stupid.”
  “You are supposed to do as much as possible in your head.”
  “Problems should come out evenly.”
  “Even if there are different ways to solve a problem, one way is best.”
  “Word problems are important in mathematics.”
  “Now I know that there are often many different ways to get an answer.”
Beliefs about self
  “I just don't have a mathematical mind.”
  “If I work hard, I can become competent in mathematics.”

(6) Attitudes about Mathematics and about Learning
Examples: test anxiety, motivation, confidence, perceived usefulness mathematics, liking/disliking mathematics in general, word problems in particular.
  “I hate estimating. I always feel stupid doing it, cause I'm always wrong.”
  “When the problem gets messy, I don't want to continue.”
  “It bothers me if I get the answer and don't know how.”
  “I feel a sense of satisfaction from my growing competence.”
4 STEPS FOR SOLVING PROBLEMS

UNDERSTANDING THE PROBLEM
Questions which can be useful to ask:
1. Do I understand what the problem is asking for?
2. Can I state the problem in my own words, that is, paraphrase the problem?
3. Have I used all the given information?
4. Can I solve a part of the problem?
Actions which can be helpful:
1. Reread the problem carefully. Often it helps to reread a problem a few times.
2. Try to use the given information to deduce more information.
3. Plug in some numbers to make the problem more concrete, more real.

DEVISING A PLAN
Here are several common strategies. Add your own strategies to the list!
1. Represent the problem with a diagram (it often helps to carefully draw and label it).
   - Check to see if you used (the relevant) given information.
   - Does the diagram "fit" the problem?
2. Think \(\rightarrow\) guess \(\rightarrow\) test \(\rightarrow\) think \(\rightarrow\) revise (vs. "groping and hoping" or "flail and wail")
   - Keep track of guesses with a table.
3. Make an estimate. A solution plan often comes from the estimation process.
4. Make a table (sometimes the key comes from adding a new column).
5. Look for patterns—in the problem or from your guesses.
6. Be systematic.
7. Look to see if the problem is similar to one already solved.
8. If the problem has "ugly" numbers, I can often "see" the problem better by substituting "nice" numbers and then go back to the original problem.
9. Make a simpler problem with smaller numbers and work my way up.
10. Break the problem down into a sequence of simpler "bite-size" problems.
11. Act it out.

CARRYING OUT THE PLAN
1. Am I keeping the problem meaningful or am I just "groping and hoping"?
   - On each step ask what the numbers mean.
   - Label my work.
2. Am I stuck and need to try another strategy?

LOOKING BACK
1. Does my answer make sense? Is the answer reasonable?
   - Is the answer close to my estimate if I made one?
2. Does my answer work when I check it with the given information?
   - (Note that checking the procedure checks the computation but not the answer.)
3. Can I use a different method to solve the problem?

This sheet is meant to serve as a starting point. The number of strategies which help the problem solving process are almost endless and vary according to each person's strengths and preferences.
After you solve a problem that was challenging for you or after you find that your answer was wrong, stop and reflect. Can you describe what you did that got you unstuck or things you did that helped you to solve the problem? If your answer was wrong, can you see what you might have done? It is the depth of these reflections that connect to your increased ability to solve problems.
Some of my basic assumptions about learning

I realize that mathematics is not everyone's favorite subject, and I realize that many students come to this course with a fair amount of trepidation and anxiety. It is important for me to reemphasize that I love teaching this course and that I enjoy working with students who find math difficult just as much as with students who love mathematics. If you are someone who has had negative experiences with mathematics in the past or someone who is not looking forward to this course, it is especially important for you to read the following. I really do believe what is written below and I have many years of experience that enables me to make these statements with confidence.

1. *Any student* can master the material in this course. Many students frequently make half-hearted attempts to read the book and half-hearted attempts to master the information. Sometimes, this behavior is due to laziness or poor study habits, but sometimes it is due to a feeling of helplessness. If you start to feel overwhelmed, please come to my office and we can address this concern together.

2. *Understanding how you got an answer* is just as important as getting the correct answer. The student whose primary focus is simply to get the right answer is often shortchanged and often does poorly on exams. This happens because the procedure or formula never made it to the student's long-term memory, because it was memorized rather than understood.

3. Practice makes perfect is only part of the statement. The whole statement should be: "*thinking while practicing* makes perfect." There is so much difference between putting the brain in gear while doing problems as opposed to mechanically practicing an algorithm that someone has shown you.

4. There are *no short-cuts*. Learning math is much like learning a language. You can't work on it once or twice a week and expect to become fluent. I just don't know of many success stories in which students did little work or only two days per week.

5. Doing mathematics can actually be *enjoyable* when you see the patterns and connections and structures which underlie mathematics!

6. Students come into the course with *different needs* and ability levels. With respect to many aspects of the course (e.g., homework, tests, etc.), there is a certain amount of flexibility. However, I need to be aware of different students' needs and difficulties.
(Note: In my courses, students turn in a portfolio at the end of the semester which includes things like: the story of the problem I am most proud of solving, some habits of mind that I have developed, some problem solving strategies/tools that I have developed, my biggest learnings of the semester. This is the handout for the habits of mind part of the portfolio.)

**Habits of mind**

There are many attributes that are associated with people who are able to learn effectively and who are able to "own" the knowledge they "learn." Here are a few key ones.

**Curiosity**
Most discoveries and inventions, scientific and artistic, have come out of curiosity, which is closely related to creativity. Curiosity looks like "What if . . .?" and “Why?” and is also related to openness—listening to others' perceptions and ideas, realizing that it is not “which way is best” but rather there are often many ways to see and solve problems. If you select curiosity, you will be able to give specific examples of "What if . . .?" and “Why?”

**Resisting impulsivity**
Some people find themselves jumping at the first idea without thinking it through more carefully; this may happen because of anxiety due to low self-confidence and it may happen because of time pressures. Another manifestation of this is called overgeneralization or oversimplification; e.g., seeing subtraction only as take-away. If you select resisting impulsivity, you will describe what impulsivity looks like in you, the factors that contribute to being impulsive, and what you have done to resist impulsivity.

**Multiple perspectives**
This involves looking at a problem/question from different perspectives--consider the image of viewing a magnificent sculpture from different angles. If you select multiple perspectives, you will describe examples of when you have seen a problem or a concept from multiple perspectives rather than narrowly; you will also describe how this habit has made you a more powerful learner.

**Open-mindedness**
Many people have a tendency to judge a new idea or strategy before fully looking at it. Many wonderful inventions and solutions to problems have come when people stayed open to a seemingly crazy idea. This quality can manifest in many ways: being open-minded to different ideas and being open-minded to ideas from other students. If you select open-mindedness, you will describe how you have become more open-minded, giving specific examples.

**Self-initiative**
Most students who have done well in this course have demonstrated this attribute. Following are a few examples of what this looks like: not only answering the problems that I assigned but extending those problems or making up similar problems; bringing in articles (from magazines or newspapers) that relate to discussions and explorations in this course; connecting work in this course to other experiences (in your schooling, in your work as a tutor, etc.). If you select self-initiative, you will give examples of self-initiative during the semester.
Characteristics of a Worthwhile Mathematical Task
(adapted from the National Council of Teachers of Mathematics)

1. It is rich—mathematically.
   a. The doing of it develops students' understandings of important ideas.
   b. It connects to other ideas in the course.
   c. It puts one or more of the process standards in the foreground—problem solving, reasoning, communication, connections, representation.
   d. It allows further challenges and/or is extendible.

2. It is rich—pedagogically
   a. The task is a genuine problem as opposed to recitation of procedure.
   b. There is not just one answer, not just one way to do it.
   c. The task involves discovery, creation.
   d. Students have some responsibility for checking their answer vs. the teacher telling them if they are right.

3. It is rich—student-wise
   a. It is interesting and “worthwhile” to the students.
   b. It honors different learning styles.
   c. Everyone can get “on the bus;” that is, it is challenging for students at different levels.

This is from a group in Michigan. A good learning task:
- engages all the senses.
- allows students to construct and explore ideas.
- has multiple paths to a valid outcome.
- is not "over engineered."
- is not rushed.
- contains sound and significant mathematics or science.

Teachers may consider the following in selecting learning tasks:
   Thinking about science or math like a scientist or mathematician will tend to encourage students to do so, too.
   Advance planning can help teachers avoid unexpected learning outcomes, but important learning can be achieved by teachers' exploration of "wrong' answers with their students.
   Tasks need not end when a class period is over. Students' work on a particular task might be sustained over a long period of time.
   Encourage students to explore; try to redirect students who request extensive detailed instructions.
Process Competencies
(adapted from the National Council of Teachers of Mathematics)

You will find that these competencies permeate most of the work we do and you will come to see the close relationship between the process competencies, habits of mind, and a deeper understanding of mathematics.

Problem Solving
You will develop various tools to help you solve problems that at first have you stumped:
• What tools have I developed to help me understand a problem that at first has me stumped?
• What problem solving tools have I added or refined?
• How have I gotten better at monitoring my progress while problem solving?
• What tools have I developed to help me to check my work?

Reasoning and proof
Many students find that "mathematical" thinking is not as alien as they thought!
• How have I gotten better at justifying my answers and solution processes?
• How have I gotten better at understanding the "why" as well as the "how"?

Communication
As mathematics and English teachers talk together, mathematics teachers find that the importance of language and notation in mathematics has been underappreciated.
• How has my appreciation of the importance and usefulness of definitions, notation, and symbols grown?

Connections
Some authors define understanding in terms of connections, i.e., understanding a new idea means being able to connect it to previously learned ideas. The focus here is on looking for connections and making connections, for example, connections within math concepts (e.g., the four meanings of fractions); connection between math concepts (e.g., between addition and subtraction); and/or connections between concepts and procedures: (e.g., why we "move over" when multiplying).

Representation
Often one representation of a problem will lead nowhere while another representation makes the solution so much easier to see!
• How have I gotten better at creating useful and effective representations to organize, record, and communicate mathematical ideas?
• How has translating among different representations of an idea deepened my understanding of that idea?

Convention
There is a world of difference between conventions (e.g., adding from left-to-right) and mathematical truths (e.g., the value of pi to two decimal places is 3.14).
• What conventions have I seen and how has understanding the difference between conventions and mathematical structure made me a more powerful learner?